NEW SUPERSPACE ACTION FOR $D=4$ SUPERSTRINGS

E. NISSIMOV [1] and S. PACHEVA [1]

Department of Physics, Weizmann Institute of Science, Rehovot 76100, Israel

Received 11 September 1989

Generalizing a previously proposed alternative $D=4$ superparticle action of a non-Brink–Schwarz type we present a new superspace action describing the dynamics of the non-compactified coordinates of space-time supersymmetric superstrings. An advantage of this new formulation over the standard Green–Schwarz one is that it allows for a straightforward manifestly $D=4$ super-Poincaré covariant quantization without the need of introducing auxiliary variables or infinite number of ghosts for ghosts. Preliminary calculations show, however, that the theory has a nonvanishing conformal anomaly if the known schemes of compactification are assumed.

1. Introduction

In the past two years much efforts were devoted to solve the problem of the manifestly super-Poincaré covariant quantization of the Green–Schwarz (GS) superstrings [1,2] in $D=10$ space–time dimensions [3–8]. The main difficulty here lies in the fact that, due to the intricate nature of the pertinent fermionic κ-gauge symmetry [9,1] (infinite stage of reducibility versus breaking of manifest Lorentz invariance), the GS superstrings cannot be quantized covariantly unless one introduces:

(i) either appropriate auxiliary variables [3–7];

(ii) or an infinite set of ghosts for ghosts [8].

In the formalism with auxiliary variables one reduces the covariantly quantized GS superstring to a free $D=2$ conformal field theory with a finite number of ghost fields [6]. However, the corresponding vertex operators [1,4] turn out to be nonpolynomial functions of the string fields which makes the task of practical amplitude computations quite difficult. On the other hand, the formalism with an infinite number of ghosts for ghosts suffers from some problems [10] and, moreover, this formalism cannot be consistently derived from the systematic Batalin–Vilkovisky procedure [11] with a finite stage of reducibility L by taking the limit $L\to\infty$. The derivation of vertex operators and amplitude calculations in the infinite ghosts-for-ghosts approach is also an open question.

The zero-mode of the GS superstring – the Brink–Schwarz superparticle [12], shares the same difficulties in the covariant quantization [13]. Since it is consistently defined in any $D=2, 3, 4 \pmod 8$ (where there exist super-Poincaré algebras) one is forced to introduce auxiliary variables even in the lower space–time dimensions $D=2, 3, 4$ (cf. ref. [14]). On the other hand, it is well known that ordinary supersymmetric field theories ($N=1$ in $D=4$, $N=1, 2$ in $D=2, 3$), some of which are second-quantized versions of the underlying BS superparticles, are consistently constructed in terms of off-shell unconstrained superfields [15] without any dependence on auxiliary variables (unlike the case of $N=2$, 3 in $D=4$ [16]).

The latter circumstance raised the possibility that alternative superparticle actions in $D=4$ of a non-Brink–Schwarz type do exist which do not need auxiliary variables (or an infinite number of ghosts for ghosts) for the sake of their super-Poincaré covariant quantization. Indeed, some time ago actions of this kind were proposed in ref. [17] within the canonical hamiltonian formalism.

In the present letter we first perform a further analysis of the $D=4$, $N=1$ superparticle action of ref.

[1] Permanent address: Institute of Nuclear Research and Nuclear Energy, Boulevard Lenin 72, 1784 Sofia, Bulgaria.

0370-2693/90/$ 03.50 © Elsevier Science Publishers B.V. (North-Holland)

[17] which describes upon second quantization the massless $D=4$ vector supermultiplet (henceforth called vector superparticle action). We show that this action is a sum of the standard BS action plus an additional term which removes the problematic fermionic κ-gauge invariance and replaces it with a bosonic gauge symmetry which turns out to be the bosonic λ-gauge symmetry of GS [1,2]. Next, we generalize the action of the vector superparticle to the superstring case. The resulting superstring action similarly consists of the standard GS action plus an additional term leaving the superstring bosonic λ-symmetry as the only *irreducible* gauge symmetry companion to the usual reparametrization- and Weyl symmetries. This new superstring action, of course, describes only the dynamics of the non-compactified superstring coordinates. The problem of finding a relevant action for the appropriate internal string degrees of freedom is beyond the scope of the present letter. In particular, it is clear that the new $D=4$ superstring action proposed below cannot arise through the standard compactification procedure applied to the usual $D=10$ GS action [18] which yields just the $D=4$ GS action for the non-compact dimensions quantizable only non-covariantly (in the light-cone gauge, unless one introduces auxiliary variables). On the other hand, since the new $D=4$ superstring action describes the massless vector supermultiplet in the point-particle limit, it is plausible to assume that it yields an alternative consistent manifestly super-Poincaré covariant description of the non-compact string degrees of freedom.

2. The vector superparticle action in $D=4$

The superparticle action describing the massless $N=1$ vector supermultiplet in $D=4$ has the following hamiltonian (phase space) form [17]:

$$S_{\mathrm{V}}=\int \mathrm{d}\tau(p_\mu\,\partial_\tau x^\mu+p_\theta^\alpha\,\partial_\tau\theta_\alpha+\bar p_{\theta\dot\alpha}\,\partial_\tau\bar\theta^{\dot\alpha}-H_{\mathrm{V}})\,, \tag{1}$$

$$H_{\mathrm{V}}\equiv\lambda p^2+\tfrac{1}{2}\mathrm{i}\chi(\mathrm{D}_\alpha\mathrm{D}^\alpha+\bar{\mathrm{D}}^{\dot\alpha}\bar{\mathrm{D}}_{\dot\alpha})\,, \tag{2}$$

where $(\theta_\alpha,\bar\theta^{\dot\alpha})$ are anticommuting Weyl spinors,

$$\mathrm{D}_\alpha\equiv-\mathrm{i}p_{\theta\alpha}-\not{p}_{\alpha\dot\beta}\bar\theta^{\dot\beta}\,,\quad \bar{\mathrm{D}}^{\dot\alpha}\equiv-\mathrm{i}p_{\bar\theta}^{\dot\alpha}-\not{p}^{\dot\alpha\beta}\theta_\beta\,, \tag{3}$$

and the Weyl-spinor indices are raised and lowered with the help of the charge conjugation matrix $C=(C^{\alpha\beta},C_{\dot\alpha\dot\beta})$. The graded Poisson brackets (PB) among the canonical variables read

$$\{p_\mu,x^\nu\}_{\mathrm{PB}}=-\delta^\nu_\mu\,,\quad \{p_\theta^\alpha,\theta_\beta\}_{\mathrm{PB}}=\delta^\alpha_\beta\,,$$
$$\{\bar p_{\theta\dot\alpha},\bar\theta^{\dot\beta}\}_{\mathrm{PB}}=\delta^{\dot\beta}_{\dot\alpha}\,, \tag{4}$$

Clearly, the fermionic functions of the phase space variables (3) are the classical counterparts of the super-covariant derivatives:

$$\mathrm{D}_\alpha=-\frac{\partial}{\partial\theta^\alpha}+\mathrm{i}\not{\partial}_{\alpha\dot\beta}\bar\theta^{\dot\beta}\,,\quad \bar{\mathrm{D}}^{\dot\alpha}=-\frac{\partial}{\partial\bar\theta_{\dot\alpha}}+\mathrm{i}\not{\partial}^{\dot\alpha\beta}\theta_\beta\,. \tag{5}$$

In eq. (2) λ and χ are bosonic Lagrange multipliers for the first-class irreducible Dirac constraints

$$\phi_0\equiv p^2=0\,,\quad \phi_1\equiv\tfrac{1}{2}\mathrm{i}(\mathrm{D}_\alpha\mathrm{D}^\alpha+\bar{\mathrm{D}}^{\dot\alpha}\bar{\mathrm{D}}_{\dot\alpha})=0\,. \tag{6a, b}$$

The corresponding Dirac constraint equations of the first-quantized theory for the superfield wave function $V=V(x,\theta,\bar\theta)$:

$$\tfrac{1}{2}\mathrm{i}(\mathrm{D}_\alpha\mathrm{D}^\alpha+\bar{\mathrm{D}}^{\dot\alpha}\bar{\mathrm{D}}_{\dot\alpha})\,V=0\,,\quad \partial^2V=0 \tag{7a, b}$$

are easily recognized as the gauge-fixing condition and the gauge-fixed equation of motion, respectively, for the free Maxwell superfield V whose gauge-fixed action reads [15] $(\mathrm{D}^2\equiv\frac{1}{2}\mathrm{D}_\alpha\mathrm{D}^\alpha,\ \bar{\mathrm{D}}^2\equiv\frac{1}{2}\bar{\mathrm{D}}^{\dot\alpha}\bar{\mathrm{D}}_{\dot\alpha})$

$$\begin{aligned}S_{\text{gauge inv}}+S_{\text{gf}}&=\tfrac{1}{8}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,V\mathrm{D}^\alpha\bar{\mathrm{D}}^2\mathrm{D}_\alpha V\\&\quad+\frac{1}{8\alpha}\int \mathrm{d}^4x\,\mathrm{d}^4\theta[(\mathrm{D}^2+\bar{\mathrm{D}}^2)\,V]^2\\&=\tfrac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4\theta\,V[\partial^2\\&\quad+(1/\alpha-1)\,\tfrac{1}{4}(\mathrm{D}^2\bar{\mathrm{D}}^2+\bar{\mathrm{D}}^2\mathrm{D}^2)]\,V\,,\end{aligned} \tag{8}$$

and provided the choice $\alpha=1$ is made.

Now, to elucidate the nature of the second bosonic gauge symmetry of the action (1) generated by the hamiltonian constraint $\mathrm{i}(\mathrm{D}^2+\bar{\mathrm{D}}^2)$ (6b), let us pass to the lagrangian formalism. Substituting the explicit expressions for D_α, $\bar{\mathrm{D}}^{\dot\alpha}$ (3) into (2) and (1) and performing straightforwardly the gaussian integrations over the canonical momenta p_μ, p_θ^α, $\bar p_{\theta\dot\alpha}$, we obtain

$$\begin{aligned}S_{\mathrm{V}}=\int \mathrm{d}\tau\bigg(&\frac{1}{4\lambda}\,[\partial_\tau x^\mu-\mathrm{i}\bar\theta^{\dot\alpha}\sigma^\mu_{\dot\alpha\beta}\partial_\tau\theta^\beta-\theta_\alpha(\sigma^\mu)^{\alpha\dot\beta}\partial_\tau\bar\theta_{\dot\beta}]^2\\&+\frac{\mathrm{i}}{2\chi}\,(\partial_\tau\theta_\alpha\,\partial_\tau\theta^\alpha+\partial_\tau\bar\theta^{\dot\alpha}\partial_\tau\bar\theta_{\dot\alpha})\bigg)\,.\end{aligned} \tag{9}$$

The first term in (9) is, of course, the usual BS action. For the second bosonic gauge symmetry we have in the hamiltonian formalism

$$\delta_{\tilde\kappa}\theta_\alpha=\{\tilde\kappa(\mathrm{D}^2+\bar{\mathrm{D}}^2),\theta_\alpha\}_{\mathrm{PB}}=-\tilde\kappa\mathrm{D}_\alpha\,,$$
$$\delta_{\tilde\kappa}\bar\theta^{\dot\alpha}=-\tilde\kappa\bar{\mathrm{D}}^{\dot\alpha}\,,$$
$$\delta_{\tilde\kappa}x^\mu=-\mathrm{i}\tilde\kappa[\bar\theta^{\dot\alpha}\sigma^\mu_{\dot\alpha\beta}\mathrm{D}^\beta+\theta_\alpha(\sigma^\mu)^{\alpha\dot\beta}\bar{\mathrm{D}}_{\dot\beta}]\,,$$
$$\delta_{\tilde\kappa}\chi=\partial_\tau\tilde\kappa\,. \tag{10}$$

Inserting into eqs. (10) the expressions for $(\mathrm{D}_\alpha, \bar{\mathrm{D}}^{\dot\alpha})$ through the velocities

$$\mathrm{D}_\alpha=-\frac{1}{\chi}\partial_\tau\theta_\alpha\,,\quad \bar{\mathrm{D}}^{\dot\alpha}=-\frac{1}{\chi}\partial_\tau\bar\theta^{\dot\alpha}\,, \tag{11}$$

and rescaling the gauge parameter $\tilde\kappa\to\kappa=\tilde\kappa/\chi$, we arrive at the following gauge transformations in the lagrangian formalism:

$$\delta_\kappa\theta_\alpha=\kappa\partial_\tau\theta_\alpha\,,\quad \delta_\kappa\bar\theta^{\dot\alpha}=\kappa\partial_\tau\bar\theta^{\dot\alpha}\,,\quad \delta_\kappa\chi=\partial_\tau(\kappa\chi)\,,$$
$$\delta_\kappa x^\mu=\mathrm{i}\kappa[\bar\theta^{\dot\alpha}\sigma^\mu_{\dot\alpha\beta}\partial_\tau\theta^\beta+\theta_\alpha(\sigma^\mu)^{\alpha\dot\beta}\partial_\tau\bar\theta_{\dot\beta}]\,. \tag{12}$$

Let us point out that both terms in the action (9) are separately invariant under the symmetry (12).

Eqs. (12) are readily recognized as the λ-symmetry transformations in the usual BS formalism [2]. There is, however, a quite significant difference in the role this symmetry plays in the standard BS and in the present framework. For the BS superparticle the relevant additional (with respect to the reparametrization invariance) gauge symmetry is the fermionic κ-gauge symmetry which reduces the number of independent θ-coordinates by half so that for $N=1$ in $D=4$ the BS action describes the *chiral* scalar supermultiplet. The κ-symmetry is generated by the irreducible first-class half of the fermionic spinor constraint $\Psi\equiv(\mathrm{D}_\alpha, \bar{\mathrm{D}}^{\dot\alpha})=0$ and, therefore, it always breaks manifest Lorentz invariance unless one introduces appropriate auxiliary variables. Thus, the λ-gauge symmetry in the BS framework is a trivial symmetry generated by a first-class combination $\mathrm{i}(\mathrm{D}^2+\bar{\mathrm{D}}^2)$ of the independent constraints $(\mathrm{D}_\alpha, \bar{\mathrm{D}}^{\dot\alpha})$. For the vector superparticle, in contrast, there is no fermionic κ-symmetry gauging away (in a non-covariant manner) part of the θ's but the constraint on the superfield wave function V (7a), imposed by the λ-symmetry, annihilates in a manifestly covariant way the chiral and the antichiral superspin zero parts of V [15] [#1].

3. Generalization to $D=4$ superstrings

To derive the superstring analogue of the vector superparticle action (9), which will describe the non-compactified superstring coordinates, it is most straightforward to start from the hamiltonian framework. Here we shall consider only the heterotic case.

From the GS formalism it is well known that the analogues of the hamiltonian first-class constraints (6a, b) read

$$\phi_0\to\mathrm{T}_{\mathrm{L,R}}(\xi)\,,\quad \phi_1\to\Delta(\xi)\,, \tag{13}$$

$$T_{\mathrm{L}}(\xi)\equiv(P_\mu-X'_\mu)^2\,, \tag{14}$$

$$T_{\mathrm{R}}(\xi)\equiv\Pi_\mu\Pi^\mu+4\mathrm{i}(\theta'^\alpha\mathrm{D}_\alpha+\bar\theta'_{\dot\alpha}\bar{\mathrm{D}}^{\dot\alpha})\equiv\Pi^2+4\mathrm{i}\bar\theta'\mathrm{D}$$
$$=(P_\mu+X'_\mu)^2+4(\theta'^\alpha p_{\theta\alpha}+\bar\theta'_{\dot\alpha}\bar p^{\dot\alpha}_\theta)\,, \tag{15}$$

$$\Delta(\xi)\equiv\tfrac12\mathrm{i}(\mathrm{D}_\alpha\mathrm{D}^\alpha+\bar{\mathrm{D}}^{\dot\alpha}\bar{\mathrm{D}}_{\dot\alpha})\equiv-\tfrac12\mathrm{i}\bar{\mathrm{D}}\mathrm{D}\,, \tag{16}$$

where the following notations are used:

$$\Pi^\mu(\xi)\equiv P^\mu+X'^\mu-2\mathrm{i}\bar\theta\gamma^\mu\theta'\,, \tag{17}$$

$$D(\xi)\equiv-\mathrm{i}C^{-1}p_\theta-(\Pi^\mu+\mathrm{i}\bar\theta\gamma^\mu\theta')(\gamma_\mu\theta)\,. \tag{18}$$

Primes denote differentiation with respect to the ξ – the space-like string world-sheet parameter. Also, from now on we shall employ for brevity the four-component Dirac (Majorana) spinor notations instead of the two-component Weyl notations.

Correspondingly, the hamiltonian form of the action reads

$$S^{\mathrm{heterotic}}=\int\mathrm{d}\tau\,\mathrm{d}\xi(P_\mu\partial_\tau X^\mu+\bar p_\theta\partial_\tau\theta$$
$$-\Lambda_{\mathrm{L}}T_{\mathrm{L}}-\Lambda_{\mathrm{R}}T_{\mathrm{R}}-\chi\Delta)\,, \tag{19}$$

where $\Lambda_{\mathrm{L,R}}$ and χ denote the bosonic Lagrange multipliers. All constraints (14)–(16) in eq. (19) are irreducible and first-class with the following PB algebra:

$$\{T_{\mathrm{L,R}}(\xi), T_{\mathrm{L,R}}(\eta)\}_{\mathrm{PB}}=\mp8[T_{\mathrm{L,R}}(\xi)\,\delta'(\xi-\eta)$$
$$+\tfrac12T'_{\mathrm{L,R}}(\xi)\,\delta(\xi-\eta)]\,, \tag{20}$$

[#1] In $D=3$ the κ- and λ-symmetries become in some sense equivalent. Both, the $D=3$ analogue of the action (9): $L=(1/4\lambda)(\partial_\tau x^\mu+\mathrm{i}\theta_\alpha\gamma^{\mu\alpha\beta}\partial_\tau\theta_\beta)^2+(\mathrm{i}/2\chi)\,\partial_\tau\theta_\alpha\partial_\tau\theta^\alpha$, as well as the ordinary $D=3$ BS action, describe the same $D=3$ scalar supermultiplet.

$$\{-\tfrac{1}{2}\mathrm{i}\bar{D}D(\xi), -\tfrac{1}{2}\mathrm{i}\bar{D}D(\eta)\}_{\mathrm{PB}} = 2\mathrm{i}\Pi^\mu(\xi)[D(\xi)(C\gamma_\mu)D(\xi)]\,\delta(\xi-\eta)=0\,, \tag{21}$$

i.e.

$$\{\Delta(\xi), \Delta(\eta)\}_{\mathrm{PB}}=0\,. \tag{22}$$

To compute (21) we have used the well-known PB for $D(\xi)$ (18):

$$\{D(\xi), D(\eta)\}_{\mathrm{PB}} = -2\mathrm{i}\Pi^\mu(\xi)(\gamma_\mu C^{-1})\,\delta(\xi-\eta)\,, \tag{23}$$

and the fact that $C\gamma_\mu$ is a symmetric 4×4 matrix whereas $D(\xi)$ (18) is an anticommuting spinor on the classical level.

Now, upon substituting the explicit expressions (14)-(18) into the action (19), one gets an expression quadratic with respect to both canonical momenta P^μ and p_θ (unlike the GS case where the dependence on p_θ is linear). After straightforward gaussian integration over P^μ and p_θ we obtain

$$S^{\text{heterotic}} = S^{\text{heterotic}}_{\text{GS}} - \frac{\mathrm{i}}{2}\int \mathrm{d}\tau\,\mathrm{d}\xi\,\sqrt{-g}\,Y^{mn}(\partial_m\bar{\theta}\partial_n\theta)\,. \tag{24}$$

Here the following notations are used:

$$S^{\text{heterotic}}_{\text{GS}} = \tfrac{1}{2}\int \mathrm{d}\tau\,\mathrm{d}\xi\,\sqrt{-g}[-g^{nm}\partial_n X^\mu \partial_m X_\mu + 4\mathrm{i}P_-^{nm}\partial_m X^\mu(\bar{\theta}\gamma_\mu\partial_n\theta) + g^{nm}(\bar{\theta}\gamma_\mu\partial_n\theta)(\bar{\theta}\gamma^\mu\partial_m\theta)] \tag{25}$$

is the ordinary heterotic GS action where the world-sheet metric g_{nm} (n, $m=0, 1$) is expressed through the Lagrange multipliers $\Lambda_{\mathrm{L,R}}$ in (19) as

$$\sqrt{-g}\,g^{00} = -[2(\Lambda_{\mathrm{L}}+\Lambda_{\mathrm{R}})]^{-1}\,,$$
$$\sqrt{-g}\,g^{01} = (\Lambda_{\mathrm{R}}-\Lambda_{\mathrm{L}})(\Lambda_{\mathrm{L}}+\Lambda_{\mathrm{R}})^{-1}\,,$$
$$\sqrt{-g}\,g^{11} = 8\Lambda_{\mathrm{L}}\Lambda_{\mathrm{R}}(\Lambda_{\mathrm{L}}+\Lambda_{\mathrm{R}})^{-1}\,. \tag{26}$$

$P_\pm^{mn}$ denote the covariant world-sheet chiral projectors

$$P_\pm^{mn} \equiv \frac{1}{2}\left(g^{mn} \pm \frac{\epsilon^{mn}}{\sqrt{-g}}\right). \tag{27}$$

In the second non-GS term of the action (24) Y^{mn} denotes a symmetric traceless tensor which is chiral with respect to both world-sheet indices:

$$P_+^{mk}Y_k^n = P_+^{nk}Y_k^m = Y^{mn}\,, \tag{28}$$

and whose sole independent component is just the inverse of the Lagrange multiplier χ from (19):

$$\frac{1}{\chi} = \sqrt{-g}\,P_+^{0m}P_+^{0n}Y_{mn}\,, \tag{29}$$

As in the superparticle case (section 2), it is straightforward to deduce in the lagrangian formalism the gauge symmetry generated by the second bosonic superstring constraint $\Delta(\xi)$ (16):

$$\delta_\kappa\theta = \kappa^m\partial_m\theta\,, \tag{30}$$

$$\delta_\kappa X^\mu = \mathrm{i}\kappa^m(\bar{\theta}\gamma_\mu\partial_m\theta)\,, \tag{31}$$

$$\delta_\kappa\left(\frac{1}{\chi}\right) = (P_+^{0l}\kappa_l)\frac{P_-^{0k}}{P_-^{00}}\partial_k\left(\frac{1}{\chi}\right) - \frac{1}{\chi}\left[\frac{P_-^{0k}}{P_-^{00}}\partial_k(P_+^{0l}\kappa_l) - \partial_k\left(\frac{P_-^{0k}}{P_-^{00}}\right)(P_+^{0l}\kappa_l)\right], \tag{32}$$

where κ^m is a self-dual world-sheet vector: $\kappa^m = P_+^{mn}\kappa_n$.

One immediately recognizes (30) and (31) as the GS bosonic λ-symmetry transformations [2].

Let us add some preliminary remarks about the quantization of the superstring action (24) within the operator (canonical quantization) formalism. For the Virasoro central charge in the supersymmetric right-moving sector one gets

$$\begin{aligned} c_{\mathrm{R}} &= 4\ (\text{from } X^\mu) + 4\times(-2)(\text{from } (\theta, p_\theta)) \\ &\quad -26\ (\text{from } (b,c) \text{ ghosts}) \\ &\quad -26\ (\text{from } (\bar{\zeta}, \zeta) \text{ ghosts}) \\ &= -56\,. \end{aligned} \tag{33}$$

In eq. (33) it is assumed that the standard separation into creation–annihilation pairs is assigned to the modes of the (θ, p_θ)-fields and the λ-symmetry ghosts $(\zeta, \bar{\zeta})$ as world-sheet fields of conformal spins (0, 1) (cf. eq. (15)) and (−1, 2), respectively. In the purely bosonic left-moving sector as usual $c_{\mathrm{L}}=4-26=-22$.

Also, due to the normal ordering in the quantum commutator of the λ-symmetry generators one gets a purely anomalous non-zero right-hand side in (22):

$$[-\tfrac{1}{2}\mathrm{i}:\bar{D}D(\xi):, -\tfrac{1}{2}\mathrm{i}:\bar{D}D(\eta):] = -\frac{4\mathrm{i}}{\pi}[:T_{\mathrm{R}}(\xi):\delta'(\xi-\eta) + \tfrac{1}{2}:T'_{\mathrm{R}}:(\xi)\,\delta(\xi-\eta) - (c_{\mathrm{R}}/12\pi)\,\delta'''(\xi-\eta)]\,. \tag{34}$$

The most immediate question is, of course, the one of how the anomalies comming from the internal superstring degrees of freedom could cancel these c-number anomalies (the conformal (33) and the λ-symmetry (34) ones). In particular, if one adds a set of non-chiral internal conformal fields with $c=22$ to cancel the left-moving conformal anomaly and part of the right-moving one, then the remaining uncompensated right-handed Virasoro charge becomes -34. It then cannot be cancelled if the compactification is achieved on a lattice [19] since the one-loop modular invariance requires that the central charge of the compactified right-moving string degrees of freedom is a multiple of 8 [#2].

A possibility of getting around this difficulty might be the existence of alternative consistent compactification schemes which do not place the kind of restrictions on the Varasoro charge as the lattice compactification schemes.

Acknowledgement

We thankfully acknowledge the cordial hospitality of the Einstein Center for Theoretical Physics and Y. Frishman at the Weizmann Institute of Science. We also thank A. Schwimmer and S. Solomon for their interest in this work.

[#2] We are indebted to A. Schwimmer for illuminating this point to us.

References

[1] M.B. Green and J.H. Schwarz, Phys. Lett. B 136 (1984) 367; Nucl. Phys. B 243 (1984) 285.

[2] M.B. Green, J.H. Schwarz and E. Witten, Superstring theory, vols. 1, 2 (Cambridge U.P., Cambridge, 1987).

[3] E. Nissimov, S. Pacheva and S. Solomon, Nucl. Phys. B 297 (1988) 349;
E. Nissimov and S. Pacheva, Phys. Lett. B 202 (1988) 325; Proc. XXI Intern. Symp. on Theory of elementary particles (Sellin, GDR, October 1981).

[4] E. Nissimov, S. Pacheva and S. Solomon, Proc. Workshop on Perspectives in string theory (Copenhagen, October 1987) (World Scientific, Singapore, 1988).

[5] E. Nissimov, S. Pacheva and S. Solomon, Nucl. Phys. B 317 (1989) 344.

[6] E. Nissimov, S. Pacheva and S. Solomon, Phys. Lett. B 228 (1989) 181; Proc. Superstring Workshop (Texas AM University, March 1989).

[7] R. Kallosh and M. Rahmanov, Phys. Lett. B 209 (1988) 233; B 214 (1988) 549.

[8] U. Lindstrom, M. Roček, W. Siegel, P. van Nieuwenhuizen and A.E. van de Ven, Phys. Lett. B 224 (1989) 285; Stony Brook preprint ITP-SB-89-38;
S. Gates et al., Phys. Lett. B 225 (1989) 44;
M.B. Green and C.M. Hull, Phys. Lett. B 225 (1989) 57;
R.E. Kallosh, Phys. Lett. B 224 (1989) 273; B 225 (1989) 49;
F. Bastianelli, G. Delius and E. Laenen, Stony Brook preprint ITP-SB-89-51;
E.A. Bergshoeff and R.E. Kallosh, CERN preprint CERN TH-5462/89.

[9] W. Siegel, Phys. Lett. B 128 (1983) 397.

[10] J. Fisch and M. Henneaux, Brussels preprint ULB TH2/89-04.

[11] I.A. Batalin and G.A. Vilkovisky, Phys. Lett. B 102 (1981) 27; Phys. Rev. D 28 (1983) 2567.

[12] R. Casalbuoni, Phys. Lett. B 62 (1976) 49; Nuovo Cimento A 33 (1976) 389;
D.V. Volkov and A.J. Pashnev, Theor. Math. Phys. 44 (1980) 321;
L. Brink and J.H. Schwarz, Phys. Lett. B 100 (1981) 310.

[13] E. Nissimov and S. Pacheva, Phys. Lett. B 189 (1987) 57;
E. Nissimov, S. Pacheva and S. Solomon, Nucl. Phys. B 296 (1988) 462; B 299 (1988) 183;
L. Brink and M. Henneaux, Principles of string theory (Plenum, New York, 1988);
L. Brink, M. Henneaux and C. Teitelboim, Nucl. Phys. B 293 (1987) 505;
M. Moshe, Proc. Johns Hopkins Workshop in Particle physics on Frontiers in particle physics (Lanzhou, China, 1987) (World Scientific, Singapore, 1988);
S. Solomon, Phys. Lett. B 203 (1988) 86;
H. Aratyn and R. Ingermanson, Phys. Rev. D 39 (1989) 503.

[14] A.K.H. Bengtsson, I. Bengtsson, M. Cederwall and N. Linden, Phys. Rev. D 36 (1987) 1766;
Y. Eisenberg and S. Solomon, Nucl. Phys. B 309 (1988) 709; Phys. Lett. B 220 (1989) 562;
D.P. Sorokin, V.I. Tkach and D.V. Volkov, Mod. Phys. Lett. A 4 (1989) 901.

[15] S. Gates, M. Grisaru, M. Roček and W. Siegel, Superspace (Benjamin/Cummings, Menlo Park, CA, 1983).

[16] A. Galperin, E. Ivanov, S. Kalitzin, V. Ogievetsky and E. Sokatchev, Class. Quantum Grav. 1 (1984) 469; 2 (1985) 155;
A. Galperin, E. Ivanov, V. Ogievetsky and E. Sokatchev, Class. Qauntum Grav. 2 (1985) 601, 617.

[17] S. Kalitzin, E. Nissimov and S. Pacheva, Mod. Phys. Lett. A 2 (1987) 651.

[18] A. Chamseddine and J.-P. Deredinger, Nucl. Phys. B 301 (1988) 381.

[19] W. Lerche, A.N. Schellekens and N.P. Warner, Phys. Rep. 177 (1989) 1.